Quiz 5: 14.6

Show all work clearly.

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(1) The temperature at a point (x, y, z) is $T(x,y,z)=10z^3e^{x^2-\frac{y^2}{4}}$ degrees Celsius where x, y and z are measured in meters. *(8points)*

Be sure to give appropriate units in answers.

(a) Find the rate of change of the temperature at (1,2,1) in the direction toward (4,3,2).

$\vec{\nabla}T = 10e^{x^2-\frac{y^2}{4}}\langle 2xz^3, -\frac{1}{2}yz^3, 3z^2\rangle$

$\vec{\nabla}T(1,2,1) = 10\langle 2,-1,3\rangle = \langle 20,-10,30\rangle$

P Q

$\vec{v} = \overrightarrow{PQ} = \langle 3,1,1\rangle$

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \left\langle \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right\rangle$

$D_{\vec{u}}T(1,2,1) = \vec{\nabla}T(1,2,1)\cdot\vec{u}$

$= \langle 20,-10,30\rangle \cdot \left\langle \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right\rangle = \frac{80}{\sqrt{11}}$ °C/m

(b) A bug at (1,2,1) wants to fly in the direction in which the temperature increases most rapidly. In what direction should the bug travel? What is the rate of increase in that direction? (units)

Temp increases in direction of $\vec{\nabla}T(1,2,1) = \langle 20,-10,30\rangle$

note: don't need to write as unit vector

Rate of increase is $\|\vec{\nabla}T(1,2,1)\| = 10\sqrt{14}$ °C/m

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(2)

(a) Find the equation of the tangent plane to the paraboloid $y = 16 - x^2 + 2z^2$ at the point (3, 7, 0).

(b) Find points, if any, on the paraboloid $y = 16 - x^2 + 2z^2$ where the tangent plane is parallel to the plane $-4x - \frac{1}{2}y + 2z = 5$.

(12 points)

a) First write plane in form $F(x,y,z) = 0$

$\underbrace{x^2+y-2z^2-16=0}_{F(x,y,z)} \Rightarrow \vec{\nabla}F = \langle 2x, 1, -4z\rangle$

Then $\vec{n} = \vec{\nabla}F(3,7,0) = \langle 6,1,0\rangle$

$\Rightarrow$ plane $6(x-3)+1(y-7)=0 \Rightarrow$ $6x+y=25$

b) Point on plane (a,b,c) such that

$\vec{\nabla}F(a,b,c) \| \langle -4,-\frac{1}{2},2\rangle$

$\langle 2a,1,-4c\rangle = k\langle -4,-\frac{1}{2},2\rangle$

$2a=-4k \quad 1=-\frac{1}{2}k \quad -4c=2k$

$2a=8 \quad K=-2 \quad -4c=-4$

$a=4 \quad c=1$

point on paraboloid $y = 16 - x^2 + 2z^2$

$b = 16-16+2$

$b=2$

$(4,2,1)$